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CONTRIBUTIONS FROM THE JEFFERSON PHYSICAL LABORATORY, HARVARD UNIVERSITY.

# *THE MAXIMUM VALUE OF THE MAGNETIZATION IN IRON.*

By B. Osgood Peirce.

With Three Plates.

# THE MAXIMUM VALUE OF THE MAGNETIZATION IN IRON.

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The first experiments on the magnetic behavior of soft iron under high excitations were made, more than sixty years ago, upon comparatively short, stout rods, so that the results were affected by the demagnetizing action of the ends of the specimen, but, even under these circumstances, several different observers [1] were able to show that if the magnetizing force to which a piece of iron is exposed be made stronger and stronger, the intensity of the resulting magnetization of the metal usually approaches a definite limit, and that this limit is practically reached in fields of such strength as are frequently used in the laboratory.

The work of Stoletow and Rowland in the early seventies of the last century, upon iron rings or toroids, made the true meanings of $H$, $B$, and $I$ in the iron clearer, and since that time many persons [2] have attempted to determine the limiting value ($I_\infty$), of $I$, as $H$ is made to increase indefinitely. $I_\infty$ is now sometimes called the *specific magnetism* of the material.

From some of his early work, to which he applied a peculiar method of extrapolation, Rowland inferred that in the case of soft iron, $I_\infty$ must be about 1390, whereas Fromme obtained the value 1510 in

[1] Joule, Phil. Mag., **2**, 1839; Mueller, Pogg. Ann. **79**, 1850; **82**, 1851; Koosen, Pogg. Ann. **85**, 1852; Dub, Pogg. Ann. **90**, 1853; G. Wiedemann, Pogg. Ann. **100**, 1851; **106**, 1859; **117**, 1862.

[2] Rowland, Phil. Mag. **46**, 1873; **48**, 1874; Stefan, Wiener Berichte, 1874; **97**, 1888. Wied. Ann. **38**, 1889; Fromme, Wied. Ann. **13**, 1881; Ewing and Low, B. A. A. S. Report 1887; Phil. Trans. **180**, 1889; H. E. J. G. duBois, Phil. Mag. **29**, 1890; Roessler, Elektrotechnische Zeitschrift, **14**, 1893; Jones, Wied. Ann. **54**, 1895; **57**, 1896; Gumlich, Elektrotechnische' Zeitschrift, **30**, 1909; Peirce, These Proceedings, **44**, 1908; Am. Journal of Science, **28**, 1909; Weiss, Journal de Physique, May, 1910; Hadfield and Hopkinson, Jour. Inst. Elect. Eng., **46**, 1911.

1873, and Stefan, 1400, in the following year. In 1881, however, Fromme got the value 1737 for one specimen, and in 1884, Weber, exposing a long rod in a solenoid to a field which had an intensity of only 900 gausses before the iron was introduced, found the corresponding value of $I$ to be 1700.

In 1887, Messrs. Ewing and Low introduced a new and most ingenious method for experimenting upon slender isthmuses of iron and steel under very high excitations and showed that different specimens of soft iron often behaved very differently in very strong fields. For one brand of fine Swedish iron, they found the final value of $I$ to be only 1620, while for a certain kind of Bessemer steel, the value $I_\infty$ was as high as 1770.

Du Bois published in 1890 the results of a series of experiments upon iron in very intense fields the strengths of which he had determined by optical means. In order to obtain, for each brand of material, the Kerr's constant which he needed, he first examined an ellipsoidal test piece of the material in much weaker fields, in a solenoid. In a typical case, the soft iron ellipsoid of revolution was 18 centimeters long and 6 millimeters in diameter at the centre. The solenoid was 30 centimeters long and consisted of 1080 turns of insulated wire wound in twelve layers of about 4 centimeters mean radius. The field intensity at a point 9 centimeters from the centre of the solenoid was about 6% less than at the centre and this introduced a correction into the formula for $I$. The moment acquired by the bar when it was under excitation was determined, after the effect of the current in the solenoid had been compensated for, by the indications of a magnetometer in Gauss's $A$ Position with respect to the specimen. The correction for the ends of the ellipsoid was made by the use of the formula $H' = H - 0.052\ I$. We shall find it convenient to refer to these results later on in this paper and some of them appear in Table I.

TABLE I.

| $H'$. | $I$. | $H'$. | $I$. |
|---|---|---|---|
| 100 | 1410 | 600 | 1680 |
| 200 | 1522 | 800 | 1695 |
| 300 | 1590 | 1000 | 1703 |
| 400 | 1630 | 1300 | 1712 |
| 500 | 1661 | | |

In the stronger fields the results were not so regular, for the specimen was magnetized between the poles of a powerful electromagnet

and the fields were far from uniform. The final value of $I$ which du Bois obtained lay somewhere between 1700 and 1750.

A paper by Roessler in the Elektrotechnische Zeitschrift for 1893 describes some experiments very like those made by du Bois with the solenoid mentioned above. Roessler's solenoid was 1 meter long and consisted of 16 layers of wire 3 millimeters in diameter. The mean radius of the solenoid was about 5.5 centimeters and the field at a point on the axis 25 centimeters from the centre was about 1% less than at the centre itself. The test piece was an ellipsoid 50 centimeters long and 1 centimeter in diameter. The results which Roessler obtained for a certain specimen of so called "soft iron" are given in Table II.

TABLE II.

| $H'$. | $I$. | $H'$. | $I$. |
|---|---|---|---|
| 414 | 1645 | 848 | 1679 |
| 481 | 1663 | 919 | 1681 |
| 516 | 1670 | 991 | 1681 |
| 587 | 1675 | 1062 | 1685 |
| 741 | 1679 | 1276 | 1683 |
| 777 | 1681 | 1312 | 1687 |

The values of $I_\infty$ published in 1896 by E. T. Jones, who magnetized a short, slender wire of the material to be tested between the conical pole pieces of an electromagnet of the du Bois form, ranged as high as 1818; and the results of the joint work of du Bois and Jones, printed in 1899, gave values of $I_\infty$ between 1780 and 1850.

Weiss, in 1907 and 1909, experimented upon small ellipsoids of revolution made of iron, nickel, and cobalt, placed symmetrically between the flat pole pieces of a powerful electromagnet. Each ellipsoid was about 9 millimeters long and 3.5 millimeters in diameter. The gap between the pole pieces was about 6 centimeters long and the diameter of the magnet core was 15 centimeters. An excitation of 94000 ampere turns corresponded to a field of about 9000 gausses in the gap centre. The small ellipsoid was suddenly drawn out of the field through a hole in the axis of one of the pole pieces and the flux change in a test solenoid outside the iron was determined. Weiss's values of $I_\infty$ were 1731 and 1706.

Gumlich in 1909 made a series of extremely accurate determinations of the final value of $I$ in soft iron by the Isthmus Method, using an electromagnet of the du Bois form, which was furnished with two soft pole pieces fastened together with the isthmus between them and

capable of being rotated together about a horizontal axis perpendicular to the pole axis, so as to reverse suddenly the sign of the magnetization in the test piece. Each specimen was 28 millimeters long and 3 millimeters in diameter. To make sure that the lines of induction in the test piece were throughout parallel to each other, Gumlich sometimes used soft iron rings slipped over the specimen. Gumlich's value of $I_\infty$ was 1725.

In December, 1910, Messrs. Hadfield and Hopkinson printed the results of a very carefully carried out and very elaborate investigation into the question whether in such combinations of iron and less magnetic substances as are in practical use, the specific magnetism of any piece of the material multiplied by the mass of the piece is simply equal to the sum of the products obtained by multiplying the mass of each constituent in the specimen by its specific magnetism. They came to the conclusion that although this rule seems not to hold in certain alloys of iron, nickel, and manganese, it is really fulfilled in many practical cases.

They used a modification of the Isthmus Method very skilfully, employing an electro-magnet like, if not identical with, the magnet which Ewing and Low had, and which was made for the first isthmus experiments, under the direction of W. Low, Esquire, of Balmakewan.

Hadfield and Hopkinson had at command a large number of alloys specially made at the Hecla Works, for research purposes, and the analyses of their test pieces are therefore beyond question. They found that in their annealed iron-carbon steel, where other elements were nearly absent, the specific magnetism was less than for their standard iron by a percentage equal to about six times the percentage of carbon. In such a case they assumed that there are two constituents, pure iron, and iron carbide ($Fe_3C$) in mechanical mixture, the percentage of the carbide present being 15.5 times that of the carbon in the steel. The "pure iron" used as a standard was a sample of Swedish iron (Maker's mark "S. C. I.") containing less than 0.2 per cent of impurities. Of this they used two specimens: one was 6.26 mm. long, and 3.18 mm. in diameter and weighed 0.385 grammes; the other was 15.92 mm. long and 3.19 mm. in diameter, and its weight was 0.99 grammes. Both yielded the same value (1680) for the specific magnetism. Table III, obtained from measurements of one of the curves given by Hadfield and Hopkinson, reproduces their results sufficiently well. Most of their pieces of steel were slightly less dense than the S. C. I. iron and their final values of $I_\infty$ give the magnetization vector per unit volume of matter of the same density as the iron.

It is now possible to get in the market large pieces of iron which has less than 0.03% of impurities all told, and I have used Norway iron

TABLE III.

HADFIELD AND HOPKINSON'S VALUES OF THE SPECIFIC MAGNETISM OF IRON AND CARBON ALLOYS.

| Percentage of Carbon. | Specific Magnetism. |
|---|---|
| * | 1680 |
| 0.5 | 1630 |
| 1.0 | 1580 |
| 1.5 | 1530 |
| 2.0 | 1480 |
| 2.5 | 1430 |
| 3.0 | 1380 |
| 3.5 | 1330 |

99.87% pure, as well as many other specimens of nearly this excellence. In some cases the specific magnetism seemed to be about 1740 and in others less than 1700. For these determinations I have usually employed some form of isthmus method.

TABLE IV.

SATURATION VALUES OF THE MAGNETIZATION VECTOR IN IRON.

| | | | |
|---|---|---|---|
| Rowland, | (1873–8) | | 1390 |
| Stefan | (1874) | | 1400 |
| Fromme | (1873) | | 1510 |
| Fromme | (1881) | | 1737 |
| Weber | (1884) | (H = 900) | 1700 |
| Ewing and Low | (1887–9) | 1620 to | 1740 |
| DuBois | (1890) | (H = 1200) | 1710 |
| Roessler | (1893) | (H = 1276) | 1688 |
| Jones | (1896) | | 1818 |
| DuBois and Jones | (1899) | 1780 to | 1850 |
| Weiss | (1907) | | 1731 |
| Gumlich | (1909) | | 1725 |
| Peirce | (1909) | 1738 to | 1751 |
| Weiss | (1910) | | 1706 |
| Hadfield and Hopkinson | (1911) | | 1680 |

If the lines of magnetic induction in a slender homogeneous cylinder, made of perfectly soft iron, are known to be straight and parallel to the generating lines of the cylinder, we may infer that the induction vector — which in this case must be solenoidal and lamellar in the metal,— has the same intensity throughout the space considered. If, moreover, the lines of force in the air about the cylinder and near it on all sides, seem to be straight, we may believe, since the tangential components of the magnetic force and the normal component of the induction are continuous at the surface of the iron, that the lines of force and induction in the metal are straight and parallel to the lines of the cylinder and to the lines just outside the metal in the air. If, therefore, by means of a test coil of very fine insulated wire wound tightly around the cylinder, and a somewhat larger coaxial coil which does not extend into any portion of the air where the lines of force are not straight, we determine $B$ in the metal and $H$ in the closely surrounding space, the ratio of the two may be supposed to give the value of the permeability in the iron. This is the theory that underlies one form of the "Isthmus Method" of measuring the value of the magnetization vector in the metal at high excitations. If the results are to be satisfactory, great care must be taken to make sure that the magnetic lines just outside the isthmus are really straight in the region to be used, and the dimensions of the test coils must be determined with the aid of trustworthy comparators with great accuracy. The larger test coil must be mounted upon some sort of support, if it is to keep its form unchanged, and the choice of material for a spool is very narrow. No brass or copper that I have ever tried is unmagnetic in very strong fields; paraffine wax and ebonite are often paramagnetic and introduce errors into the readings. Silk insulation for the wire of which the test coils are made is inadmissible without careful examination and even shellack, when dried from an alcoholic solution, is almost always strongly magnetic.

The form of bobbin used by Ewing and Low requires a fresh outer test coil for each specimen, but the little rods inserted at the ends into holes in the pole pieces, as in the work of Gumlich, or the shorter rods butted against the faces of the pole pieces, as in the work of Hadfield and Hopkinson, do not have this disadvantage.

If the lines of force in the air about the isthmus are practically straight for one excitation they often cease to be so when the intensity of the field is much increased. If, with soft iron pole pieces the lines are parallel for a soft iron bobbin, they may not be even approximately so for a bobbin of fairly hard steel, as I have found to my cost in a somewhat large experience.

These and other difficulties lie in the way of anyone who attempts to use the Isthmus Method in its original form, and every modification of it, whatever advantages it has, usually introduces some new problems. Notwithstanding all this, the method is a most useful one and has a much wider application than has usually been given it. No other way that has been proposed of making magnetic measurements at very high excitations is nearly so good, and the test pieces now employed are small and convenient to make.

There is still in many cases some uncertainty in the determinaton of $H$, and Hadfield and Hopkinson discuss the subject in trying to account for the differences between their results[3] and those of Gumlich, obtained at the Reichsanstalt. Moreover, there is sometimes irregularity in the values of $I$ measured by the Isthmus Method[4] for a single specimen. For these reasons, it has seemed to me worth while to push the use of the solenoid for magnetizing test pieces farther than has yet been done, to make sure that there are specimens of metal in which $I$ is higher than 1700 even in much weaker fields than those which the Isthmus Method furnishes. This is especially desirable if we wish to be able to determine the constitution of a large mass of steel by a quick measurement of the specific magnetism of a small test piece in an electromagnet arranged for the purpose, as has been proposed.

According to the molecular theory of magnetization of Weber and Ewing, the molecules, which lie with their magnetic axes in all directions when the metal is in the neutral state, tend to turn in the direction of any magnetic field to which the iron may be exposed, though they are hindered from doing so by the interaction of the molecules themselves. When, however, the applied field is made strong enough to overcome these intermolecular forces, in large measure, all the axes of the elementary magnets point practically in the same direction. It is evident, therefore, that unless the applied field affects the moments of the elementary magnets of which the metal is made up, the magnetic moment ($I$) of the metal per unit volume should remain nearly constant after the excitation has gone beyond a certain large value. This maximum magnetization is very different in different metals and we may well consider it as characteristic of a material.

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3 Journal of the Institution of Electrical Engineers, Dec., 1910, p. 253.
4 Ewing's Magnetic Induction in Iron and Other Metals, Tables XI, XII.

## Apparatus and Method of Procedure.

Figure 1 shows diagrammatically the general arrangement of the great array of apparatus used in making the measurements described in this paper. This apparatus was adjusted and some of it constructed

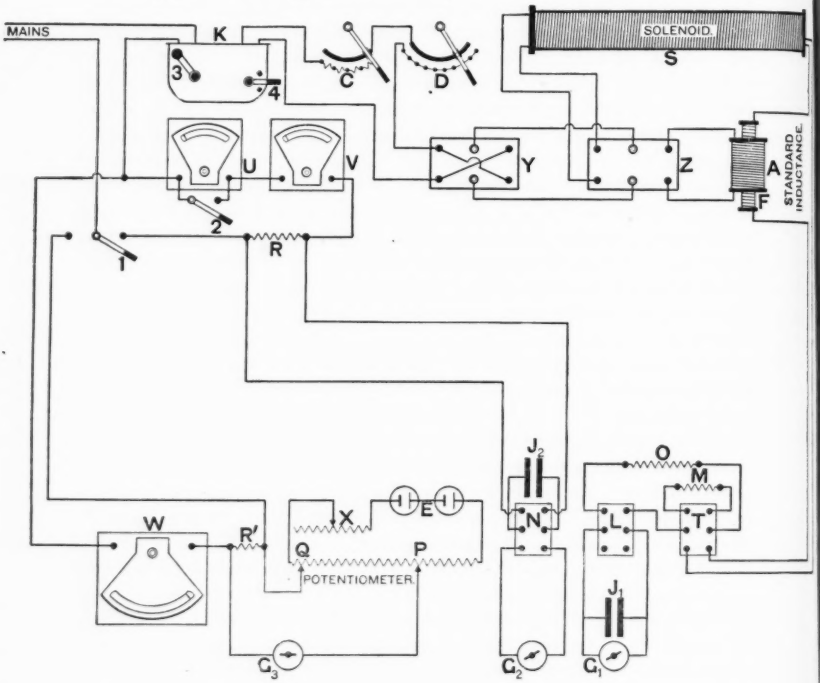


Figure 1. This Figure shows diagrammatically the general arrangement of some of the apparatus used in making the observations described in this paper. The elaborate devices for demagnetizing the specimens are omitted for simplicity.

by Mr. John Coulson of the scientific staff of the Jefferson Laboratory, who has worked with me at every stage of the investigation, and to whose skill and patience I am deeply indebted. Many details are omitted from the figure. The devices for demagnetizing the specimens to be tested will be described later on.

It is evident that in such measurements of magnetic flux changes as are necessary in the work described in this paper, it is of fundamental importance that the ballistic galvanometers used be correctly calibrated, and we used a number of standards of mutual inductance, most of them rather larger than those commonly employed for such purposes,

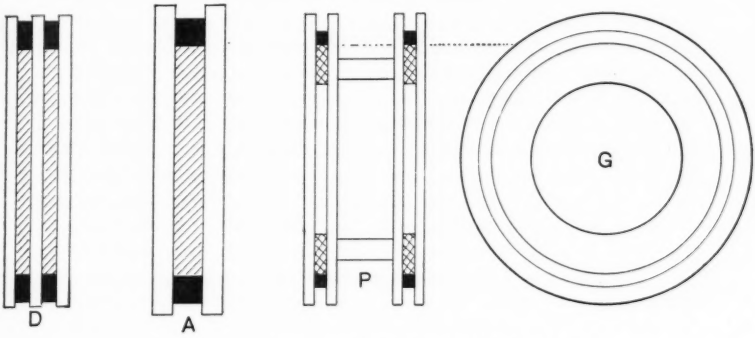


FIGURE 2. Three standards of mutual inductance.

since our rather slowly moving galvanometer was not very sensitive. We had in all seventeen mutual inductances for our calibrations. Of these five have been measured for us this year at the United States Bureau of Standards, and seven others are of such forms that their values may be calculated by well known methods. We found one of Doctor Campbell's Variable Standards of Mutual Inductance (which was very kindly lent to us by Professor Kennelly), most useful. It proved to be very accurately calibrated, and it agreed closely at all points with the standards determined for us at Washington.

An elaborate series of comparisons of our inductances occupied Mr. Coulson and myself for more than two months, because we found that three or four of those which, according to our computations based upon their geometrical forms, should have certain values, seemed to have slightly different values, though they did not seem to be quite constant. This phenomenon puzzled us at first and gave us much trouble, but we believe, after all our work, that the ebonite used as a core in three of them is very slightly susceptible in a strong magnetic field, that the split thick brass tube used as a core for one of our solenoids is sufficiently paramagnetic to affect the field inside it perceptibly, though another solenoid constructed in a similar manner seems

free from this difficulty, and finally, that the white silk triple covering of some of our wire is hygroscopic and that when a closely wound coil of it is damp, there may be a very little leakage from turn to turn through the insulation, under very strong excitation. In any event, we have eliminated all error from these sources, and we believe that the inductances of the standards we have finally used may be depended upon to at least the twentieth of one per cent.

The shapes of three of our standards are shown in Figure 2. In D, the three larger plates (and the shaded cores) are of plate glass about 29 centimeters in diameter. The cores were mounted by Mr. Thompson in an engine lathe, and were ground for about two days, under a constant flow of soda water, by a rapidly turning carborundum wheel fastened to the tool post and driven by its own motor, while the lathe moved slowly. In this way the plates were made very accurately circular. A is also wound upon a plate glass spool, but the two coils are wound together from two spools of wire triply covered with white silk. P consists of two spools with plate glass ends, but the shaded cores are ebonite rings. G shows a side view of P.

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The magnetizing solenoid consists of about 300 kilograms of triply covered Number 10 copper wire wound uniformly with great care, by Mr. George W. Thompson, upon a massive brass spool 186.2 centimeters long in inside measurement. The inner coil has 8117 turns in 14 layers, and a resistance at room temperatures of about 7.7 ohms. The outer coil, of slightly different wire, has 5872 turns in 10 layers, and a resistance of about 9.8 ohms. The field intensity at different points of the axis was found for a given current in each layer separately, and it appeared from combining the results, that a current of one ampere sent through the whole inner coil gives rise to a field of intensity 54.71(3) gausses at the centre and 54.60(5) gausses at a point 50 centimeters from the centre, on the axis. A current of one ampere sent through both coils in series creates an electromagnetic field of intensity 94.19(5) gausses at the centre and 93.77(5) gausses at a point 50 centimeters from the centre, a difference of nearly one half percent. The outside diameter of the solenoid is a little less than 20 centimeters.

For currents up to 31.5 amperes, corresponding to a field of about 2900 gausses, the coils were used in series attached to the 550 volt circuit of the Harvard University plant. For stronger fields of 5000 gausses or more, the coils could be attached in parallel to this circuit

with a standard amperemeter in each branch. For the preliminary experiments, fields stronger than 4600 gausses were not needed.

The thick, solid-drawn brass tube upon which the wire was wound carried a stream of tap water to keep the specimen at a constant temperature. The test coil was wound upon the test piece after the latter had received a very thin film of varnish. The test coil, after it had been made, was varnished and the whole was then placed for about half an hour in a stream of hot air to harden the coating. The leads were enclosed in a very thin tube of rubber, the test coil was covered with a rubber shield, and melted paraffine wax was then run into the ends of this shield so as to keep the test coil absolutely dry. In this manner all leakage from turn to turn of the triply silk covered wire of which the test coil was made was avoided. In many cases two test coils were wound side by side upon each specimen, but the results, after we had learned how to make the coils properly, were so nearly identical for both coils that we sometimes used only one. In all cases the differences, if there ever were any real differences, were far smaller than the unavoidable errors of ballistic galvanometer reading.

The reversal of a strong current in the circuit of a solenoid with so great an inductance as this one, has to be managed carefully. The main reversing switch, when it was slightly pulled, automatically put the solenoid in parallel with a noninductively wound resistance higher than its own, and, after the handle was raised higher, broke the main circuit so that the discharge from the solenoid could pass through the auxiliary resistance. The process was inverted when the switch handle was pushed down. This switch (Figure 3, Plate 1) was designed and made by Mr. Coulson.

To prove that the field in the solenoid, when a given current passes through the circuit, is really what it should be, according to the calculation, a very carefully made test coil without iron was placed in series with the secondary of a standard of mutual inductance and the field was thus measured. By this means it was shown that there was no appreciable leakage between the turns of the solenoid — a very common fault of the exciting coils of electromagnets — and that there was not enough iron in the brass of the reel to affect the field strength sensibly. So far as we can determine the fact by our many and repeated tests, the solenoid has not been injured by use and is very perfect. It is firmly mounted on a solid oak frame so that its axis is horizontal and perpendicular to the meridian.

The dimensions of the iron test pieces and of the standard induc-

tances were obtained with the help of a set of micrometer screw gauges by Brown and Sharpe. The smallest one of these was used for determining the diameters of the specimens and of the coils wound upon them. The accuracy of this gauge was tested by a comparator (which had a screw by Gaertner), and by another comparator (by Zeiss) which reads directly to microns.

An illustration, the case of a specimen of American Ingot Iron, will show how much error would be introduced into the value of the specific magnetism of the iron by a given error in measurement of its dimensions. The length was 100 cm., the diameter of the bare iron, 1.278(5) cm. and the mean diameter of the test coil, 1.326 cm. The coil consisted of 100 turns of copper wire triply covered with white silk, and as the dimensions show, the flux through it was $128.32\ B + 9.8\ H$. The last term which shows the correction for the air flux linked with the coil, is relatively small at feeble excitations, and even when $H$ rises to 2800 and $B$ becomes about 24500, the whole term is less than 1% of the flux through the iron. Moreover the value of the term may be found to within one twenty-fifth of its value without trouble. An error of 0.001 in measuring the diameter of the iron might make an error of three units in the last place in the value of the specific magnetism and this makes it desirable to use exactly round rods. The piece here described was cut out of a large bar with great skill, at the works of Messrs. Barbour and Stockwell.

At high excitations, the corrections for the effect of the ends of the cylindrical test pieces are, of course, much less than those which according to theory and to the formulas of DuBois and of Shuddemagen are necessary in low fields. I shall hope to discuss this matter at length in another paper, and need only state here that the correction for a piece of the dimensions used was practically negligible in fields of strength above 2000 gausses.

The rods to be annealed were first packed tightly in fine iron filings in a piece of pipe the ends of which were closed by screw caps, and the whole was carefully supported perpendicular to the meridian in a special gas heater where it would be exposed to several hundred flames driven by a power compressor. In this manner a piece 150 centimeters long could be heated very uniformly. After the specimen had been kept for perhaps an hour at a temperature considerably above the critical point of the iron it could be then allowed to cool very slowly *in situ*, protected from magnetic action.

If a slender rod of iron be placed inside a long solenoid which is in the secondary circuit of a powerful open-core transformer, and if

while the primary circuit is attached to the alternate current mains, the secondary coil of the transformer be slowly drawn off the core and the primary coil, by help of some mechanical device, it is possible to send through the solenoid a long series of currents alternating in direction and gradually decreasing in intensity and thus to demagnetize the iron rod very well. We had an apparatus of this kind permanently connected with our apparatus, but it was not shown in Figure 1 lest the diagram be too complex.

When the direction of a strong electrical current in the circuit of the large solenoid ($S$) in which the iron rods to be tested were magnetized, was suddenly reversed, some time was needed to establish the new current in its full value, and the change in the magnetic flux through the test coils wound upon the rods was not complete until after several seconds. This fact, due to the large inductance in the circuit, made it unsafe to employ a ballistic galvanometer of ordinary type for measuring this flux change, and we had recourse to a long period instrument of a kind which has been used for a number of years in the Jefferson Laboratory. The particular galvanometer ($G$) we chose, had a period of 156 seconds which was quite long enough for our purposes, but we had a much more slowly moving instrument at hand in case of need. Any fairly long throw of $G$ could be determined with an error of less than one tenth of one per cent, and we could do better than this by careful repetition. $G$ is shown in Figure 4, Plate 1.

The main currents in the solenoid circuit were measured with the help of a series of Weston Amperemeters (two of which are shown diagrammatically as $U$ and $V$ in Figure 1) properly arranged for the special intensity ranges we needed, but the accurate determination of large currents was made by aid of a potentiometer (Figure 5) with standard cadmium cells, which measured the potential drop across a standard one hundredth of an ohm resistance ($R$) by Crompton, which had been tested against another standard by Wolff. The largest currents we used could not very well be allowed to run very long through the coils because the amount of heat set free in the circuit was enormous. Indeed, with an energy expenditure of more than fifty kilowatts, the heating problem, in spite of running water in the core of the solenoid needed careful consideration. As a matter of fact, the only difficulty we finally encountered was a slight falling off of our largest currents with repeated throws, owing to a little increase in the resistance of the circuit, and this came at a place where the flow of inductance through the test coil changed very slowly with $H$. To save time we arranged a standard condenser (Elliot Brothers,

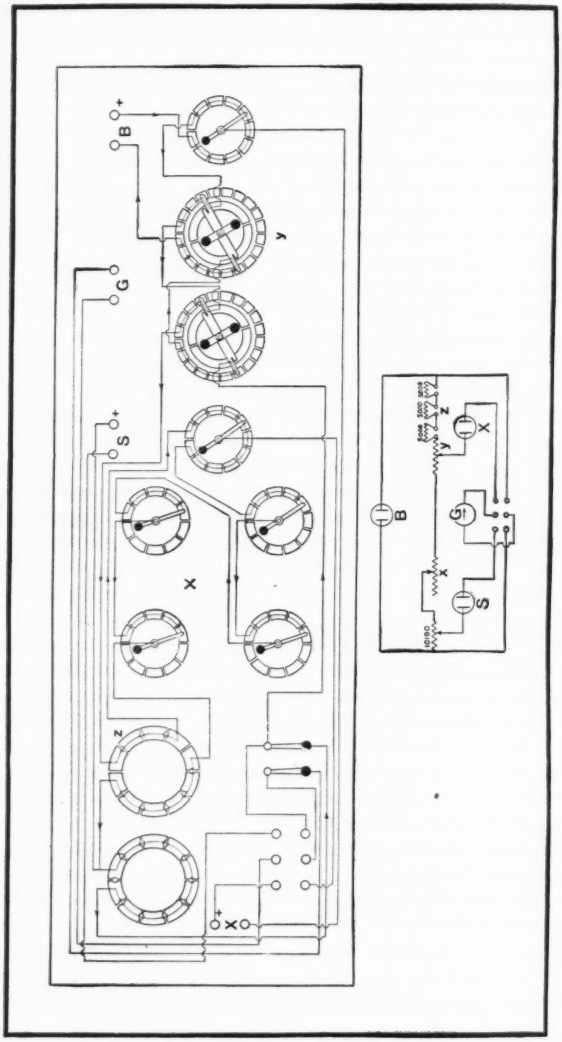


FIGURE 5. The standard potentiometer.

No. 72) so that it became automatically charged at the terminals of $R$ just as the main switch was reversed, and the charge could then be measured at our ease four or five seconds after the switch had been thrown over. By these means we avoided the delay which would have resulted if we had been obliged to read the amperemeters before the reversal.

Besides this slowly moving ballistic galvanometer, we used three other mirror galvanometers, one for the condenser throws, one for the potentiometer, and one for the accurate comparison of our inductances, and in addition, a large standard laboratory amperemeter ($W$), by Weston, which could be checked at any instant against the potentiometer. This beautiful instrument has an engine divided scale 31 cms. long.

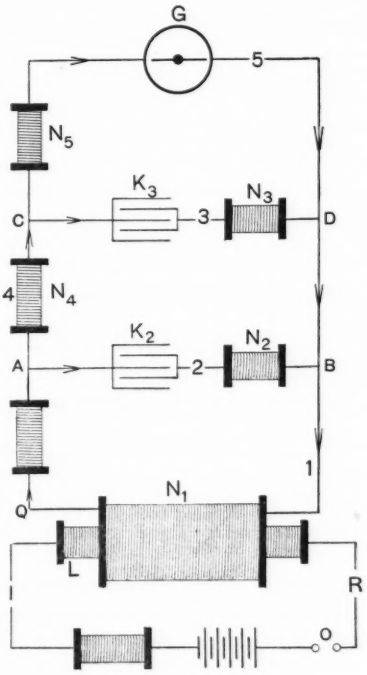

FIGURE 6.

At very high excitations, the reversal of any switch of ordinary construction gives rise to a very unpleasant explosion, and we often made use of a large controller ($K$) constructed by the General Electric Company for use upon electric cars. This was very kindly lent to us by Mr. F. W. Lieberknecht, and served an excellent purpose. We do not need to describe a large number of auxiliary amperemeters and galvanometers used in our work.

## THE USE OF CONDENSERS IN THE INDUCTIVE SECONDARY CIRCUIT.

The inductance in the secondary circuit, which contained the test coil or coils, the secondary coils of the inductance standards, and the

coil of the large ballistic galvanometer, was usually considerable and the strain upon the insulation of the wire was sometimes large when a powerful current in the primary circuit was suddenly reversed. Occasionally, there seemed to be some leakage in this circuit, so we introduced a number of condensers into the circuit in the attempt to reduce

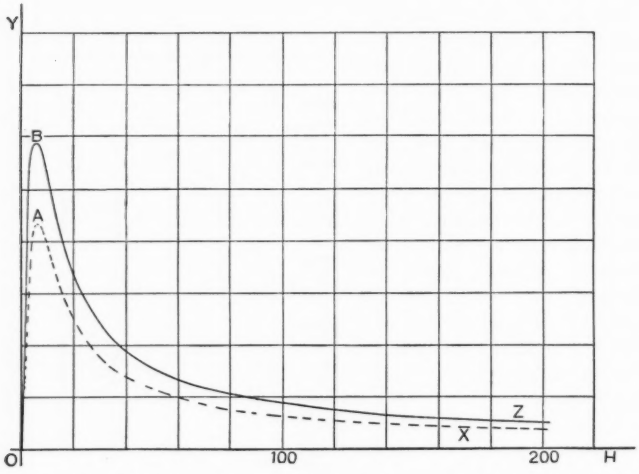


FIGURE 7. This figure shows the forms of the curves obtained by plotting the permeability and the susceptibility of a certain kind of soft iron against the exciting field. These curves, OBZ, OAX, are drawn to different scales.

the stress. What the exact effect of such condensers in a complex circuit will be when the breaking arc in the primary circuit is oscillatory, it is usually impossible to predict, because some necessary data are wanting or because the literal equations are of too high a degree to be solved, but certain general facts are clear. The following analysis treats some questions, as applied to a circuit taken for illustration, which are really, perhaps, too elementary to need any discussion.

Figure 6 represents two neighboring circuits:—

(*a*) A primary circuit of total resistance $R$, and total self inductance $L$, which contains a constant battery of voltage, $V$, and carries a current $I$. This circuit is furnished with a gap (O) which may be closed or opened at pleasure.

(*b*) A secondary circuit of several branches, which has no battery,

but which is linked with the primary circuit by the mutual inductance, $M$. The branch AQB or (1), which is directly coupled with the primary, has a resistance $R_1$, a total self inductance $N_1$, and carries a current $I_1$. The branches AB, CD, or (2), (3), contain condensers of capacities $K_2$ and $K_3$ respectively. Their resistances are $R_2$ and $R_3$, their self inductances, $N_2$ and $N_3$ and they carry currents $I_2$ and $I_3$. The branches (4) and (5) have no condensers. Their resistances are $R_4$ and $R_5$, their self inductances, $N_4$, $N_5$, and their currents, $I_4$ and $I_5$. The current in DB, which has a negligible resistance, is, of course, $I_4$.

If accents are used to denote differentiations with respect to the time, an easy application of Kirchhoff's Laws to these two circuits leads to the equations:—

$$
\begin{aligned}
&V - L\cdot I' - M\cdot I_1' = R\cdot I,\\
&-M\cdot I' - N_1\cdot I_1' - N_4\cdot I_4' - N_5\cdot I_5'\\
&\qquad = R_1\cdot I_1 + R_4\cdot I_4 + R_5\cdot I_5,\\
&-M\cdot I' - N_1(I_2' + I_3' + I_5') - N_2\cdot I_2' - Q_2/K_2\\
&\qquad = R_1(I_2 + I_3 + I_5) + R_2\cdot I_2,\\
&-M\cdot I' - N_1(I_2' + I_3' + I_5') - N_4(I_3' + I_5') - N_3\cdot I_3' - Q_3/K_3\\
&\qquad = R_1(I_2 + I_3 + I_5) + R_4(I_3 + I_5) + R_3\cdot I_3,\\
&I_1 = I_2 + I_4 = I_2 + I_3 + I_5,
\end{aligned}
\tag{1}
$$

or

$$
\begin{aligned}
&(L\cdot I' + R\cdot I) + M\cdot I_2' + M\cdot I_3' + M\cdot I_5' = V,\\
&M\cdot I' + (N_1\cdot I_2' + R_1\cdot I_2) + (N_1\cdot I_3' + N_4\cdot I_3' + R_1\cdot I_3 + R_4\cdot I_3)\\
&\quad + (N_1\cdot I_5' + N_4\cdot I_5' + N_5\cdot I_5' + R_1\cdot I_5 + R_4\cdot I_5 + R_5\cdot I_5) = 0,\\
&M\cdot I'' + (N_1\cdot I_2'' + N_2\cdot I_2'' + R_1\cdot I_2' + R_2\cdot I_2' + I_2/K_2)\\
&\qquad + (N_1\cdot I_3'' + R_1 I_3') + (N_1\cdot I_5'' + R_1\cdot I_5') = 0,\\
&M\cdot I'' + (N_1\cdot I_2'' + R_1\cdot I_2') + (N_1\cdot I_3'' + N_4\cdot I_3'' + N_3\cdot I_3''\\
&\qquad + R_1\cdot I_3' + R_4\cdot I_3' + R_3\cdot I_3' + I_3/K_3)\\
&\qquad + (N_1\cdot I_5'' + N_4\cdot I_5'' + R_1\cdot I_5' + R_4\cdot I_5') = 0.
\end{aligned}
\tag{2}
$$

If, for $I$ we write $I_0 + V/R$, the second number of the first equation becomes zero, while all the equations remain otherwise unchanged in form, and it follows that every one of the currents satisfies a single linear differential equation of the sixth order with constant coefficients

and that if $a$, $b$, $c$, $d$, $e$, and $h$ are the roots of the equation formed by equating to zero the determinant,

$$\begin{vmatrix} Lx+r & Mx & Mx & Mx \\ Mx & N_1x+R_1 & \{N_1x+N_4x+R_1 \\ & & \quad +R_4\} & \{N_1x+N_4x+N_5x \\ & & & \quad +R_1+R_4+R_5\} \\ Mx^2 & \{N_1x^2+N_2x^2+R_1x \\ & \quad +R_2x+1/K_2\} & N_1x^2+R_1x & N_1x^2+R_1 \\ Mx^2 & N_1x^2+R_1x & \{N_1x^2+N_4x^2+N_3x^2 \\ & & \quad +R_1x+R_4x+R_3x \\ & & \quad +1/K_3\} & \{N_1x^2+N_4x^2+R_4x \\ & & & \quad +R_4x\} \end{vmatrix} \tag{3}$$

then

$$\begin{aligned} I &= Ae^{at} + Be^{bt} + Ce^{ct} + De^{dt} + Ee^{et} + He^{ht} + \frac{V}{R} \\ I_2 &= \alpha_2 Ae^{at} + \beta_2 Be^{bt} + \gamma_2 Ce^{ct} + \delta_2 De^{dt} + \epsilon_2 Ee^{et} + \eta_2 He^{ht} \\ I_3 &= \alpha_3 Ae^{at} + \beta_3 Be^{bt} + \gamma_3 Ce^{ct} + \delta_3 De^{dt} + \epsilon_3 Ee^{et} + \eta_3 He^{ht} \\ I_5 &= \alpha_5 Ae^{at} + \beta_5 Be^{bt} + \gamma_5 Ce^{ct} + \delta_5 De^{dt} + \epsilon_5 Ee^{et} + \eta_5 He^{ht} \end{aligned} \tag{4}$$

If these values be substituted in one of the Kirchhoff equations above and the coefficients of the different exponential expressions separately equated to zero, it will appear that the $\alpha$'s, $\beta$'s, $\gamma$'s, $\delta$'s, $\epsilon$'s, and $\eta$'s are determinate functions of the constants of the circuit and in no way dependent upon the manner in which the currents are managed.

The other six constants ($A$, $B$, $C$, $D$, $E$, $H$) have to be computed from a knowledge of the electrical conditions which determine any problem concerning these two fixed circuits.

If, for instance, there is no current in any branch of the circuits at the outset, and if the gap, O, be suddenly closed at the origin of time, the values of the constants for all positive time satisfy the equations

$$\begin{aligned} A + B + C + D + E + H &= -\frac{V}{R} \\ \alpha_2 A + \beta_2 B + \gamma_2 C + \delta_2 D + \epsilon_2 E + \epsilon_2 H &= 0 \\ \alpha_3 A + \beta_3 B + \gamma_3 C + \delta_3 D + \epsilon_3 E + \epsilon_3 H &= 0 \\ \alpha_5 A + \beta_5 B + \gamma_5 C + \delta_5 D + \epsilon_5 E + \epsilon_5 H &= 0 \end{aligned} \tag{5}$$

and, after these have been solved, it is easy to compute the whole flow of electricity through the galvanometer, for

$$\int_0^\infty I_5\, dt = -\left(\frac{\alpha_5 A}{a} + \frac{\beta_5 B}{b} + \frac{\gamma_4 C}{c} + \frac{\delta_5 D}{d} + \frac{\epsilon_5 E}{e} + \frac{\eta_5 H}{h}\right) \tag{6}$$

If, however, when the primary current has its steady value, $V/R$, and there are no currents in the secondary circuit, the primary resistance be instantaneously changed from $R$ to $R'$, at the time $t = 0$, there is no sudden change in the current in any branch, but for all subsequent time, the constants are determined by the equations:—

$$\begin{aligned} A + \ B + \ C + \ D + \ E + \ H &= \frac{V}{R} - \frac{V}{R'} \\ \alpha_2 A + \beta_2 B + \gamma_2 C + \delta_2 D + \epsilon_2 E + \eta_2 H &= 0 \\ \alpha_3 A + \beta_3 B + \gamma_3 C + \delta_3 D + \epsilon_3 E + \eta_3 H &= 0 \\ \alpha_5 A + \beta_5 B + \gamma_5 C + \delta_5 D + \epsilon_5 E + \eta_5 H &= 0 \end{aligned} \tag{7}$$

and it is evident that every one of the quantities, $A, B, C, D, E, H, I$, given by (7) has a value which bears to the corresponding value given by (5) the ratio $(R - R')/R'$, and the same relation holds between the whole discharges through the galvanometer in the two cases. If the gap be instantly opened so that $R'$ is infinite, when the current in the primary circuit is $V/R$ and there are no secondary currents, the galvanometer throw is equal, but opposite in sign, to the throw caused by suddenly closing the gap when all the currents are zero.

The electrokinetic energy for the coupled circuits is

$$\begin{aligned} T = {} & \tfrac{1}{2}L \cdot I^2 + M \cdot I(I_2 + I_3 + I_5) + \tfrac{1}{2}N_1(I_2 + I_3 + I_5)^2 \\ & + \tfrac{1}{2}N_2 \cdot I_2^2 + \tfrac{1}{2}N_3 \cdot I_3^2 + \tfrac{1}{2}N_4(I_3 + I_5)^2 + \tfrac{1}{2}N_5 \cdot I_5, \end{aligned} \tag{8}$$

so that the electrokinetic momenta are

$$\begin{aligned} p &= L \cdot I + M(I_2 + I_3 + I_5) \\ p_2 &= M \cdot I + N_1(I_2 + I_3 + I_5) + N_2 \cdot I_2 \\ p_3 &= M \cdot I + N_1(I_2 + I_3 + I_5) + N_3 \cdot I_3 + N_4(I_3 + I_5) \\ p_5 &= M \cdot I + N_1(I_2 + I_3 + I_5) + N_4(I_3 + I_5) + N_5 I_5 \end{aligned} \tag{9}$$

If, when $I$ has the value $I = V/R$, and there are no other currents, the gap be instantly opened, $I$ suddenly drops to zero, and $I_2$, $I_3$, $I_5$, which were 0, suddenly acquire initial values which may be determined by the fact that the electrokinetic momenta, $p_2$, $p_3$, $p_5$, which before the change were equal to $P_0 = MV/R$, are not altered by the impulse. After the gap is opened, the currents in the branches obey the system of equations (2), but the initial values of these currents are to be found from the equations

$$\begin{aligned} (N_1 + N_2)I_2 + N_1I_3 + N_1I_5 &= P_0 \\ N_1I_2 + (N_1 + N_3 + N_4)I_3 + (N_1 + N_4)I_5 &= P_0 \\ N_1I_2 + (N_1 + N_4)I_3 + (N_1 + N_4 + N_5)\,I_5 &= P_0 \end{aligned} \qquad (10)$$

If then $\Delta$ denote the determinant of the coefficients,

$$\Delta = N_1N_2N_3 + N_1N_3N_4 + N_1N_3N_5 + N_1N_2N_5 + N_1N_4N_5 + N_2N_3N_5 + N_2N_3N_4 + N_2N_4N_5,$$

and the values of $I_2$, $I_3$ and $I_5$ just after the gap is opened, are

$$(N_3N_4 + N_3N_5 + N_4N_5)P_0/\Delta,\ N_2N_5P_0/\Delta,\ \text{and}\ N_2N_3P_0/\Delta. \qquad (11)$$

The total amount of electricity carried by the currents $I_2$, $I_3$, and $I_5$ are 0, 0, and $\Omega$; and to find $\Omega$ we may integrate the second equation of the system (2) with respect to the time from 0 to $\infty$ and use the initial values of $I_2$, $I_3$, $I_5$ just found. This procedure leads to the equation:

$$\Omega = \int_0^\infty I_5 \cdot dt = \frac{P_0}{R_1 + R_4 + R_5}, \qquad (12)$$

and this is evidently the same result that would have been obtained for the whole discharge through the galvanometer, if the branches (2) and (3), with their condensers, were removed from the secondary circuit. It is easy to compute the sudden loss of energy when the gap is opened.

## Results.

For the purposes of the investigation here described, we used about twenty-five different brands of iron obtained from several different sources. Of these, five gave values of $I$ larger than 1700 for comparatively low excitations of about 2800 gausses.

About ten of our specimens were described by the dealers as "Bessemer" and showed similar micrographs. Most of these were in no way remarkable. For excitations of about 2700 they gave values of $I$ of about 1675 in the average and might be expected to give 1685 for fields of strength 5000. One specimen (No. 10) was quite different from the others. For $H = 2730$ the corresponding value of $I$ was 1727. This result is based upon several different determinations made upon different days, and during the interval the rod was once annealed. A long series of annealings however reduced the permeability so that

finally the $I$ corresponding to $H = 2600$ fell to about 1700. One specimen of wrought iron which we have annealed a great number of times shows no permanent change in permeability although at one stage this fell by more than one per cent temporarily, and was restored by the next annealing.

According to my experience during the last few years with a good many pieces of so called "Norway Iron," about one specimen in three of those bought without care in the open market may be expected to have a specific magnetism considerably above 1700. Different portions of the same large bar may have very different permeabilities, however, as one may readily believe after an examination of a series of micrographs which always show a considerable amount of slag. I believe that an occasional small piece such as would be used for an isthmus might be found to have a specific magnetism three or four percent above the best value to be found in a bar. I have myself encountered two isthmuses which gave 1790 and 1751 respectively, in spite of my best efforts to reduce what seemed to me at the time impossibly large values. Some small specimens used by other observers have shown even greater values than this. In the case of a rod a meter long and twelve millimeters in diameter, however, I have never found an average value much above 1740.

Much of the wrought iron to be had in the market, though very useful to blacksmiths, contains such an amount of slag that the continuity of the metal is seriously affected and the permeability of the mass is not very high. Such are the specimens of "Farnley Iron," marked here "F," the "Taylor Iron" and the "Best Refined Iron," which show low values of $I$ in moderate fields. It is possible to get in the open market, "Norway Iron" of great purity. One specimen which I used showed, upon analysis, no nickel, cobalt, manganese or tungsten. It contained less than 0.03% of carbon, less than 0.047% of phosphorus, less than 0.03% of silicon and less than 0.003% of sulphur. This however does not compare in purity with the "American Ingot Iron," which contains less than 0.03% of impurities all told, and shows a very remarkable micrograph.

Our specimens of this iron were very kindly furnished by Doctor Percy W. Bridgman, who has been using this material in some of his experiments upon the behavior of metals under very high pressures. Plate II shows micrographs of two pieces. The first was in the normal state: the second had been exposed by Dr. Bridgman to a hydrostatic pressure of 17000 atmospheres for about 16 hours! The magnification is 120 diameters.

TABLE V.

VALUES OF THE MAGNETIZATION VECTOR IN VARIOUS KINDS OF IRON.

| No. | Material. | Diameter. | *H* | *I* |
|---|---|---|---|---|
| 1 | "Taylor Iron" | 0.975 | 2490 | 1645 |
| 2 | "Taylor Iron" | 0.998 | 2685 | 1654 |
| 3 | Bessemer | 1.266 | 2695 | 1663 |
| 4 | Bessemer | 1.269 | 2675 | 1687 |
| 5 | Bessemer | 1.269 | 2790 | 1677 |
| 6 | Bessemer | 0.950 | 2880 | 1671 |
| 7 | Bessemer | 0.950 | 2825 | 1666 |
| 8 | Bessemer | 1.269 | 2370 | 1685 |
| 9 | Bessemer | 0.634 | 2800 | 1673 |
| 10 | Bessemer | 0.632 | 2730 | 1727 |
| 11 | "American Ingot Iron" | 1.277 | 2760 | 1708 |
| 11 | "American Ingot Iron" | 1.277 | 4395 | 1711 |
| 12 | "American Ingot Iron" | 1.279 | 2780 | 1725 |
| 12 | "American Ingot Iron" | 1.279 | 4545 | 1735 |
| 13 | Norway | 1.078 | 2865 | 1686 |
| 14 | Norway | 0.969 | 2885 | 1685 |
| 15 | Norway | 1.302 | 2770 | 1661 |
| 16 | Norway | 1.280 | 2770 | 1727 |
| 16 | Norway | 1.280 | 4360 | 1742 |
| 17 | Norway | 0.294 | 2970 | 1735 |
| 18 | "Cold Rolled Shafting" | 1.269 | 2680 | 1693 |
| 19 | "Cold Rolled Shafting" | 1.269 | 2740 | 1678 |
| 20 | "Best Refined Iron" | 1.280 | 2740 | 1620 |
| 21 | "Best Refined Iron" | 0.999 | 2825 | 1589 |
| 22 | "Best Refined Iron" | 1.002 | 2690 | 1611 |
| 23 | Drill Rod | 0.794 | 2790 | 1533 |
| 24 | "R" | | 2810 | 1637 |
| 25 | "F" | 0.968 | 2665 | 1627 |

Mr. Herbert M. Boylston, of Messrs. Sauveur & Boylston, who has most kindly examined, under the microscope, the polished and etched specimens of these irons, reports that Nos. 1, 2, 11, 12, 14, 20, 21, 22, and 25 contain little, if any carbon. Of the Bessemers, with which "R" must be reckoned, No. 6 contains only about 0.05% of carbon, while No. 10 which has a high specific magnetism has 0.15% and

each of the other pieces about 0.10%. The Drill Rod has about 1.10% of carbon; the specimen, which is of very fine grain, has no slag and shows simply sorbite, pearlite, and some cementite in fine net work.

Nos. 1, 2, 20, 21, 22, 24, and 25 contain considerable slag, and this is in comparatively large masses in portions of the "Refined Iron. The relatively small amount of slag in the Norway Irons is in fine particles distributed through the mass. Nos. 11, and 12 seem to be simply ferrite.

Table VI gives some corresponding values of $H$ and $I$ for the specimen of American Ingot Iron, known as No. 12.

TABLE VI.

| *H.* | *I.* | *H/I.* |
|---|---|---|
| 40.5 | 1343 | 0.030 |
| 68.8 | 1398 | 0.049 |
| 129.9 | 1476 | 0.088 |
| 229.8 | 1570 | 0.146 |
| 271 | 1597 | 0.170 |
| 399 | 1660 | 0.240 |
| 457 | 1684 | 0.271 |
| 871 | 1715 | 0.507 |
| 1126 | 1717 | 0.655 |
| 1569 | 1719 | 0.913 |
| 2288 | 1724 | 1.323 |
| 2747 | 1727 | 1.589 |
| 4543 | 1735 | 2.620 |

It is well known that if a relatively stout rod of soft iron be exposed to a strong field in a solenoid and if the magnetizing current be very suddenly broken, the direction of the residual magnetism in the rod may be opposite in sign to what it was when the current was running. If the rod be enclosed in a thick walled copper tube within the solenoid, this reversal never takes place and the sign of the residual magnetism is always normal. The moment of the rod under the new field is often larger when the current is suddenly reversed than when it is slowly reduced to zero through a constantly growing resistance before the switch is thrown over and then gradually brought to its new strength, or when the change is made less violent by eddy currents induced in a thick copper shell around the specimen. Though it seemed possible that the results given in this paper might be slightly

affected by this so-called von Waltenhofen phenomenon, we could not discover the least difference in our results whether the rod to be used was or was not surrounded by a thick copper tube, though the tube makes the throws a trifle more regular.

It is well known, also, that under low excitations, the magnetic moment acquired by a rod in a solenoid under a given final excitation may be much increased if the rod be constantly tapped while the magnetic changes are taking place. So far as we can make out this effect is entirely lacking at very high excitations. We used a large electric tapping apparatus made by Mr. Coulson to give many sharp blows per second to a brass rod butted upon the specimen in the solenoid, and Table VII shows characteristic results.

TABLE VII.

| Current. | Flux Change when the Iron was undisturbed. | Flux Change when the Iron was tapped. |
|---|---|---|
| 4.85 | 939a | 938a |
| 8.95 | 974a | 974a |
| 30.2 | 1013a | 1013a |

Some years ago I encountered three specimens of very pure Norway Iron, each of which showed a very high specific magnetism, when tested by a modification of the Isthmus Method. Each piece was about 8 cms. long and 1.26 cms. in diameter. They were presumably from different sources.

TABLE VIII.

| Specimen. | Exciting Field. | $I$. |
|---|---|---|
| 1 | 2500 | 1733 |
| 2 | 2400 | 1738 |
| 3 | 2350 | 1751 |

Each of these numbers comes from a series of closely consistent values for different field strengths, and I have no reason to think that the determinations were not good, but I consider the probable error somewhat greater in all work I have done with isthmuses than with such experiments as I have made with larger specimens, with the help of a solenoid.

A single slender isthmus cut from the bar from which No. 2 in this table was taken, gave the very large value 1796 for $I$ in fields above 6000, but other larger pieces from the same bar showed lower values for $I$. According to my experience very small bits taken from

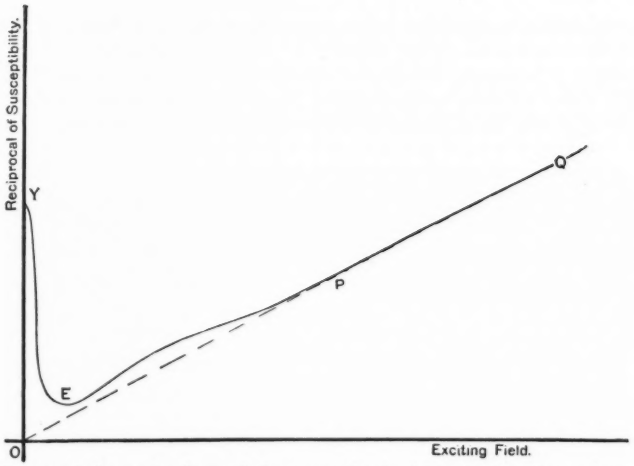


FIGURE 8 shows the form of the curve obtained by plotting the reciprocal of the susceptibility of soft iron against the intensity of the exciting field. The ratio of the abscissa of any point of the curve to the corresponding ordinate is less than the final value of $I$, and the tangent of the angle which the tangent to the curve makes with the ordinate axis is greater than this value except for small values of $H$.

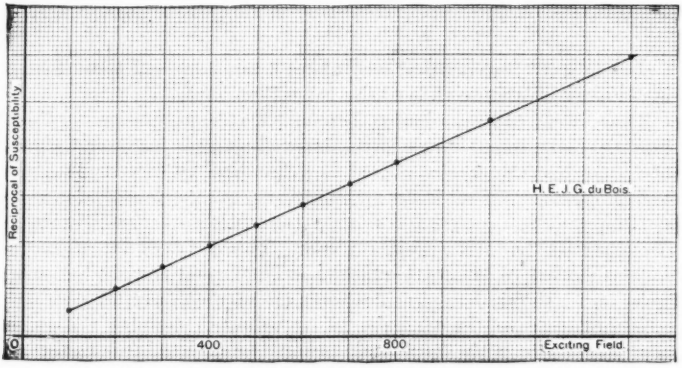


FIGURE 9. This curve shows the results of observations made by DuBois upon an ellipsoidal piece of soft iron 18 cms. long and 0.6 cms. in diameter at the middle.

closely adjacent regions of the same bar may have very different specific magnetisms, owing perhaps to differently arranged inclusions of slag. It is certain that pieces cut across the direction of the rolling often show different permeabilities from those of pieces cut in the same region in the direction of the length of the bar. Gumlich found

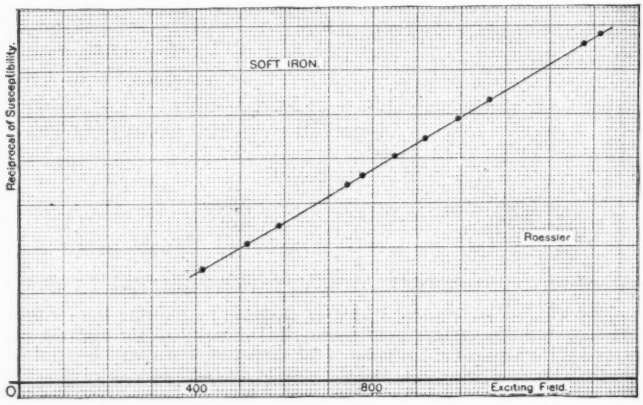


FIGURE 10. This Figure shows the results of observations made by Roessler upon an ellipsoidal piece of soft iron 50 cms. long and 1 cm. in diameter at the center.

a piece of soft "Steirisches Eisen" about 3 cms. long and about 3 mm. in diameter which also showed the value 1796 for $I$.

There seems to be no doubt, therefore, that some specimens of soft iron are to be found which have materially higher maximum values of $I$ than had the specimen used as a standard by Messrs. Hadfield and Hopkinson. Four different observers, using solenoids for magnetizing their test pieces, and seven persons using other methods have thought that they met with such pieces. This fact does not, of course, make the work of Messrs. Hadfield and Hopkinson any the less valuable, but it shows, I think, since some pieces which contain considerable quantities of $Fe_3C$, have given values of $I$ above 1720, that material bought in the open market cannot be expected to obey the law which the series of steel pieces from the Hecla Works follow.

Still, the majority, perhaps, of pieces of iron and steel bought at random will have specific magnetisms not very different from the

values given as a result of experiments upon these beautiful test pieces.

If a series of observations be made by the Method of Reversals, upon a piece of iron originally in a neutral state, and if the permeability and the susceptibility obtained in this way be plotted against the

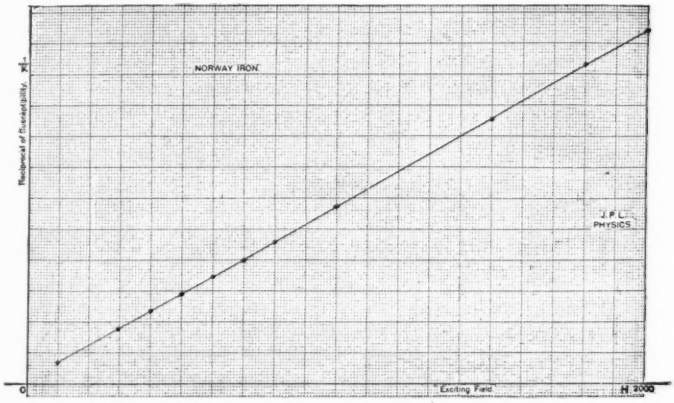


FIGURE 11 shows the results of observations made in the Jefferson Laboratory upon a rod of Norway Iron. For excitations up to about 400, the specimen was magnetized in a solenoid. For more intense fields, the determinations were made by a modification of the Isthmus Method.

intensity of the exciting field, those portions of the resulting curves (Figure 7) which correspond to large values of $H$ resemble hyperbolas which have the $x$ and the $y$ axes as asymptotes. A generation ago, therefore, it seems to have occurred to a number of persons at about the same time, that if the reciprocals of the permeability and of the susceptibility were plotted against $H$, the curves must become finally more or less straight. It appeared upon trial that for values of $H$ larger than 100, say, the reluctance gives a line only slightly convex upwards, and that the reciprocal of the susceptibility which for comparatively weak fields has the general shape shown in Figure 8 becomes very nearly coincident with a straight line drawn through the origin under high excitation. This last function has been found useful by Professor Kennelly in his paper upon the relation between

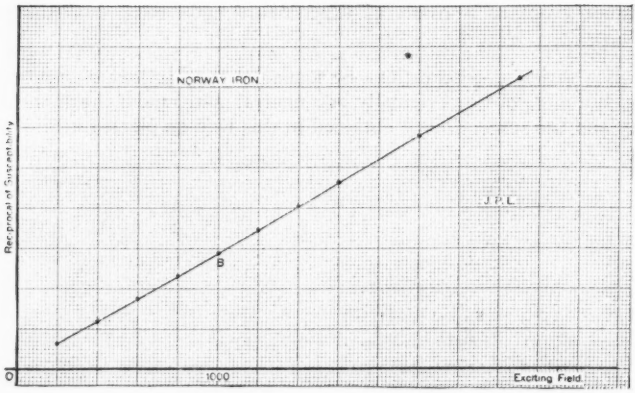


FIGURE 12 represents observations made in the Jefferson Laboratory upon a second specimen of Norway Iron.

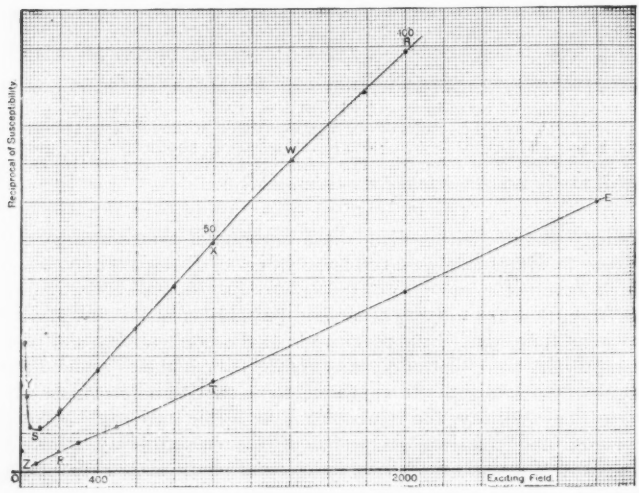


FIGURE 13. This Figure shows results obtained from tests made upon a specimen of Bessemer steel, 8.0 cms. long and 1.26 cms. in diameter. For low excitations the tests were made in a long slender solenoid. For higher fields a modification of the Isthmus Method was used.

$B$ and $H$ in fields [5] of commercial strength. It is clear from the curve in Figure 8 the ordinates of which are $H/I = 1/k$, that the ratio of the abscissa of any point of the curve to its ordinate always yields a value of $I$ somewhat less than the saturation value, whereas the slope against the ordinate axis of the tangent of the curve, after $H$

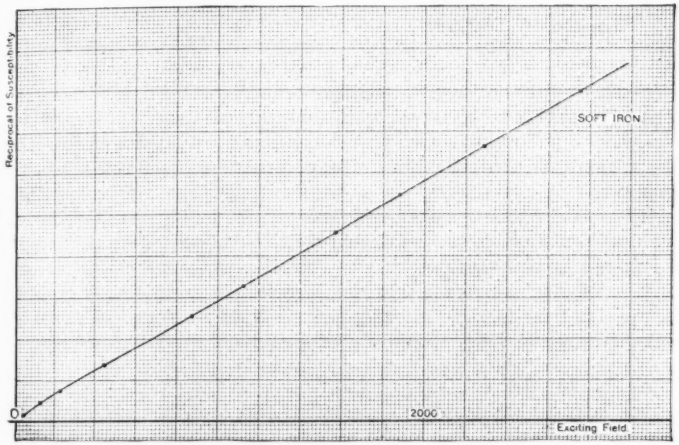


FIGURE 14. This Figure is plotted form the observations made in the Jefferson Laboratory upon a piece of "American Ingot Iron" magnetized in the solenoid. The piece was 100 cms. long and 1.279 cms. in diameter.

equals perhaps 200, is always greater than $I$. Such curves as this are especially useful when one wishes to study the saturation values of the magnetization in iron or steel.

Figures 9 and 10 show the results of plotting the reciprocals of the susceptibilities obtained by DuBois, and Roessler in their experiments already described.

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[5] Lenz and Jacobi, Pogg. Ann. **47**, 1839; Mueller, Pogg. Ann. **79**, 1850; Von Waltenhofen, Wiener Berichte, **52**, 1865; Lamont, Handbuch d. Magnetismus, p. 41; Sohncke, Elektrotechnische Zeitschrift, 1883; Ruths, Ueber d. Magnetismus weicher Eisenxylinder, 1876; Froelich, Elektrotechnische Zeitschrift, 1881, 1882, 1894; Kennelly, Trans. Am. Inst. El. Eng. **8**; Steinmetz, Elektrotechnische Zeitschrift, **13**, 1892; Muellendorf, Elektrotechnische Zeitschrift, **22**, 1901; **23**, 1902; Kapp, Electrician, **18**, 1886; Winkelmann's Handbuch der Physik, V, p. 200; Walter, Drude Ann. **14**, 1904; Czermak and Hausmaninger, Wiener Berichte, **98**, 1889; Du Bois, Wied. Ann. **51**, 1894; Fromme, Wied. Ann. **13**, 1881; **33**, 1888.

Figures 11, 12 and 13 reproduce the results of a series of measurements made two or three years ago in the Jefferson Laboratory upon specimens of Bessemer steel and of Norway Iron. For excitations up to about 400 the specimens were magnetized in a slender solenoid about five meters long, but for stronger fields a modification of the Isthmus Method was employed. Figure 14 shows some measurements made lately upon a specimen of American Ingot Iron magnetized in the shorter solenoid described above. Such curves become practically straight for much weaker fields in the case of some irons than in others.

I wish to express my great obligation to the Trustees of the Bache Fund of the National Academy of Sciences for the loan of some of the apparatus used in making the observations mentioned in this paper.

THE JEFFERSON PHYSICAL LABORATORY,
CAMBRIDGE, MASS.

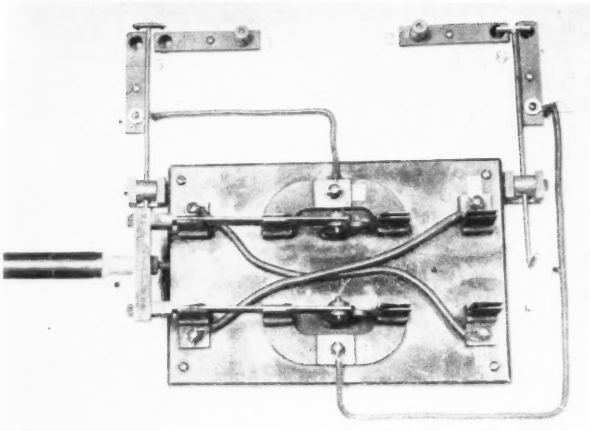


Figure 3. A switch in the main circuit so arranged that if it be suddenly thrown over, the energy in the medium which accompanied the old current is spent largely in heating an auxiliary coil.

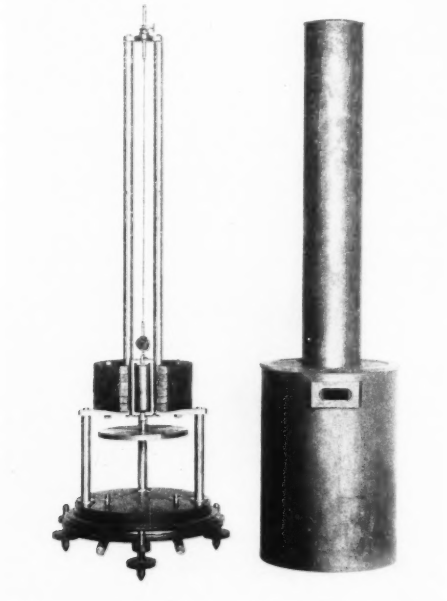
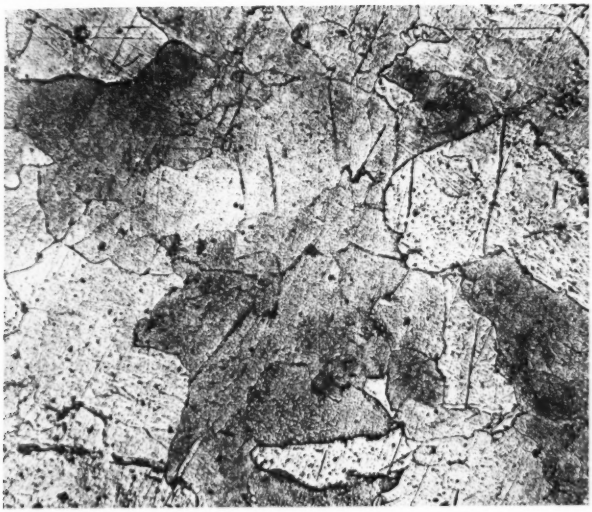
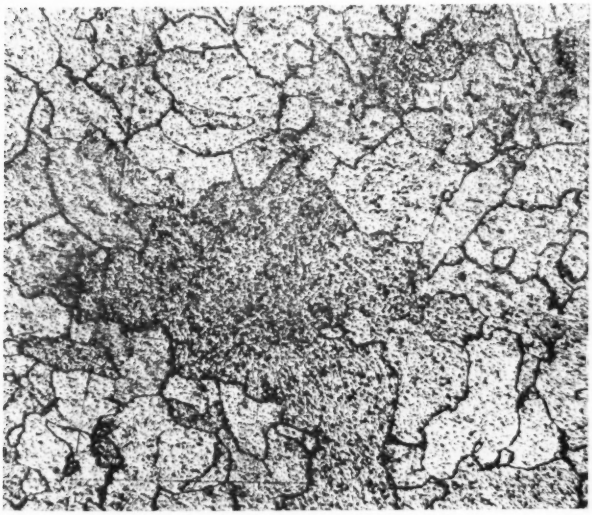
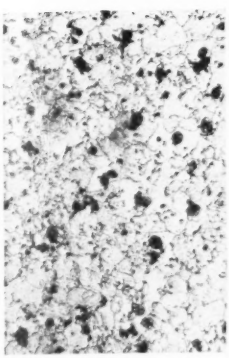


Figure 4. The long period ballistic galvanometer.





*Magnification 120.*



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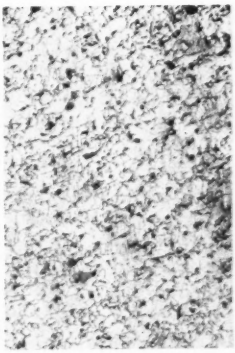

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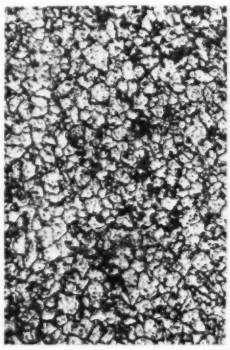

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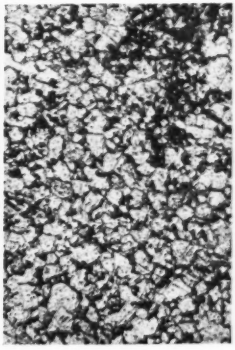

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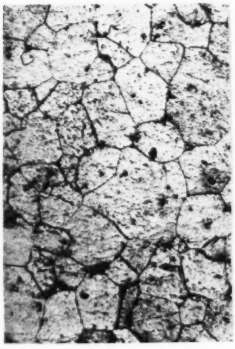

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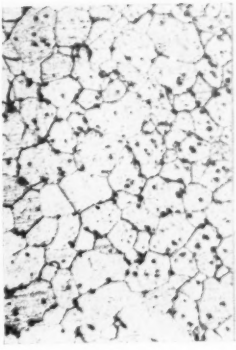

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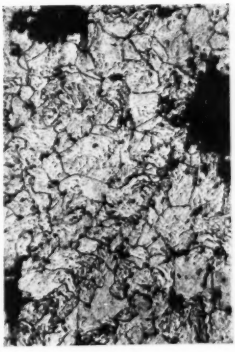

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*Magnification 120.*